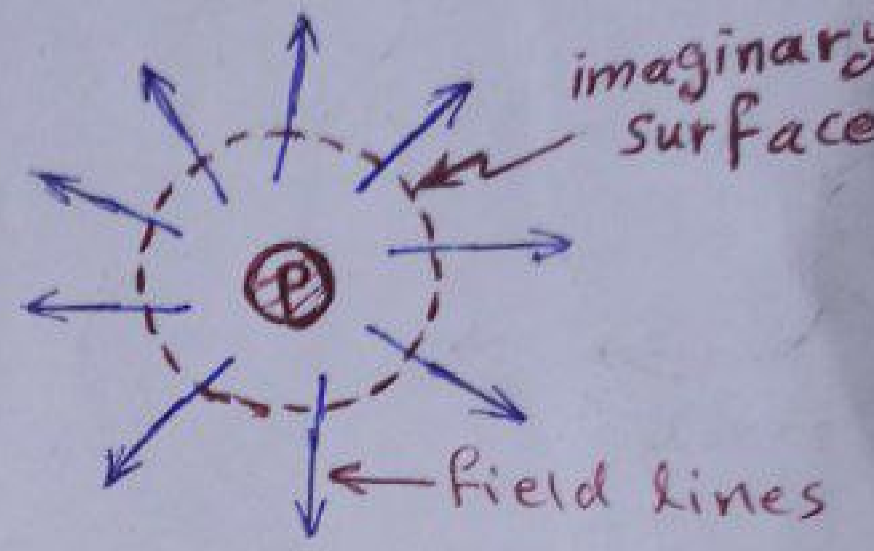
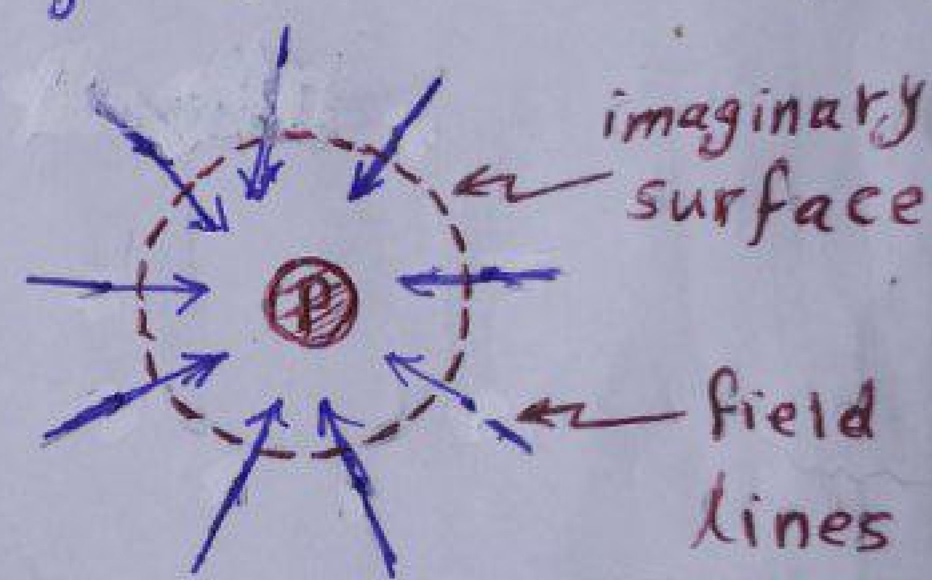
Divergence ($\nabla \cdot$)

The divergence operator (or del operator) is a measure of the vector flow (out of or into) imaginary surface surrunding point in free space.

- If the flowing or spreading is out of imaginary surface, the divergence is positive & point (P) act as a source, as shown in figure.

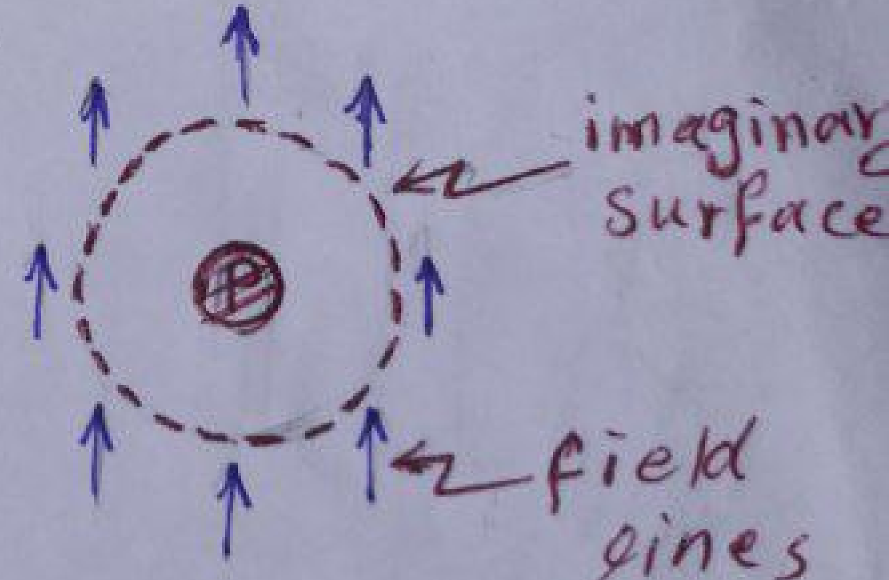


- If the flowing or spreading is toward the surface, the divergence is negative & the point (P) act as a sink as shown in the figure.

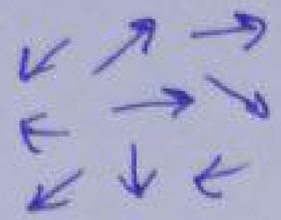


- If the outflow is equal to the inflow, the divergence is zero.

In other word, in the top of figure, there is a flow out from the imaginary surface equals to the inflow at the bottom.



* $\nabla \cdot$: means dot product & the o/p of this operator is scalar (ex// $\nabla \cdot \vec{E} = \rho / \epsilon_0$)

Divergence	any vector field	scalar field
$\vec{\nabla}\cdot$		2 4 1 0 3 5 9 6 7

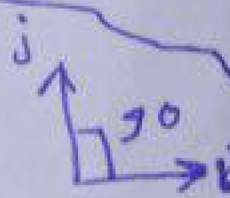
math. example

If $\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$ } ← (gradient) spatial deravation

& $\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$

so, $\vec{\nabla}\cdot\vec{E} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\cdot\left(E_x\vec{i} + E_y\vec{j} + E_z\vec{k}\right)$

$\vec{i}\cdot\vec{j} = 0$



where

$\vec{i}\cdot\vec{j} = i\cdot j\cos(90) = 0$

while

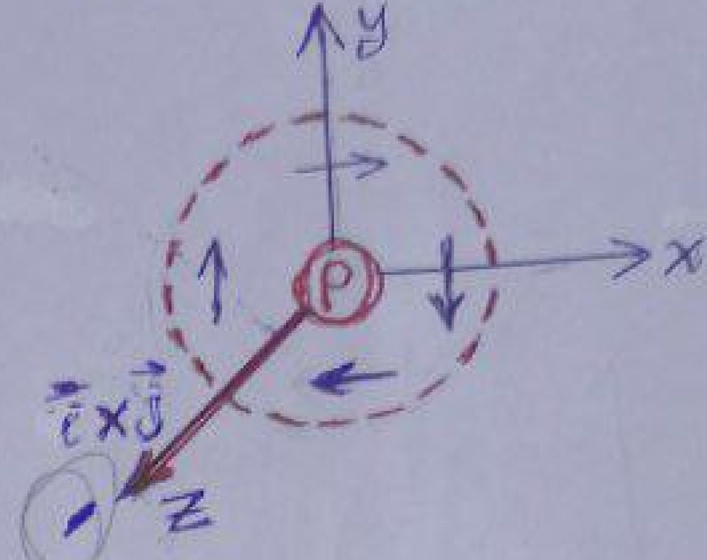
$i\cdot i = j\cdot j = k\cdot k = 1$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \frac{\partial}{\partial x}(E_x) + \frac{\partial}{\partial y}(E_y) + \frac{\partial}{\partial z}(E_z)}$$

* The output is scalar (there is no vectors)
* The flow in 3-dimention (x, y, z)

Curl ($\nabla \times$)

The curl operator is a measure of the rotation or circulation of a vector field.

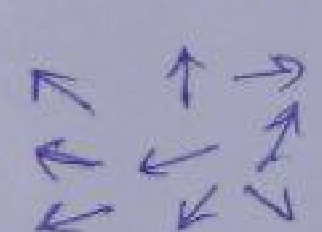


- The curl is not zero, if the rotation do effect on the point which is rotating around it.

- If the rotation in x-y plane is counterclockwise, by using the right hand rule (thumb point in the +z-direction, the hand will curl around the axis of propagation in the positive direction), that is, the curl is positive & denoted by ⊙

- If the rotation in the clockwise direction (thumb points in the −z-direction), that is, the curl is negative & denoted by ⊗ as seen in figure

* $\nabla \times$: mean cross product & the o/p of this operation is vector (ex// $\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$).

curl $\nabla \times$	vector field 	vector field 

Maxwell's equations

To understand the world, we must understand what equations mean, and not just know mathmatical constructs.

Maxwell's equations are laws - just like the law of gravity. These equations are rules the universe uses to govern the behavior of electric and magnetec fields.

In general, A flow of electric current will produce a magnetic field. If the current flow varies with time (as in any wave or periodic signal), the magnetic field will also give rise to an electric field. Maxwell's equations shows that seprated charge (positive & negative) gives rise to an electric field - and if this is varying in time (acceleration) as well will give rise to a propagating electric field, further giving rise to a propagating magnetic field.

So, the maxwell's equations are:

Divergems equations

1- $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

2- $\vec{\nabla} \cdot \vec{B} = 0$

- independent of time.
- the o/p is scalar.

Curl equations

3- $\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$

4- $\vec{\nabla} \times \vec{B} = -\frac{\partial}{\partial t} \vec{E} + J$

- dependent of time.
- the o/p is vector

* From equations, we see that maxwell's equations contain four variables which are

E - Electric field

B - Magnetic field

ρ - electric charge density

J - electric current density

$\boxed{1 - \vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0}$ means,

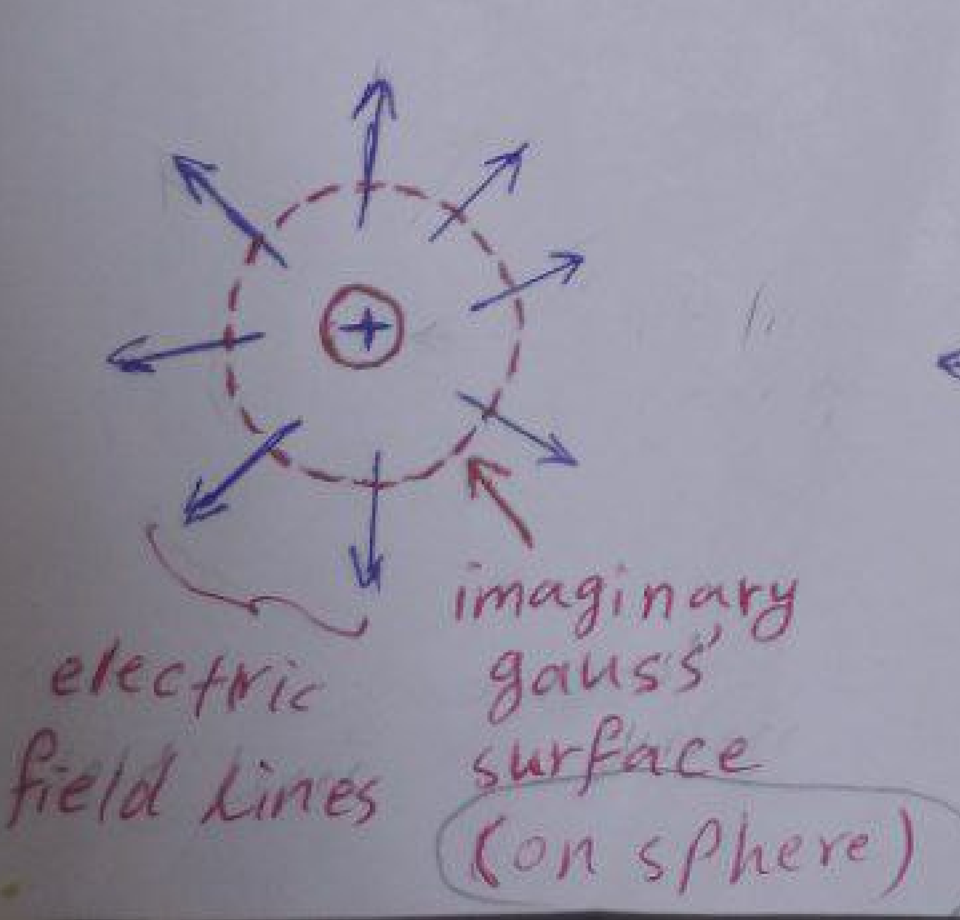
$\boxed{\nabla \cdot}$ means source

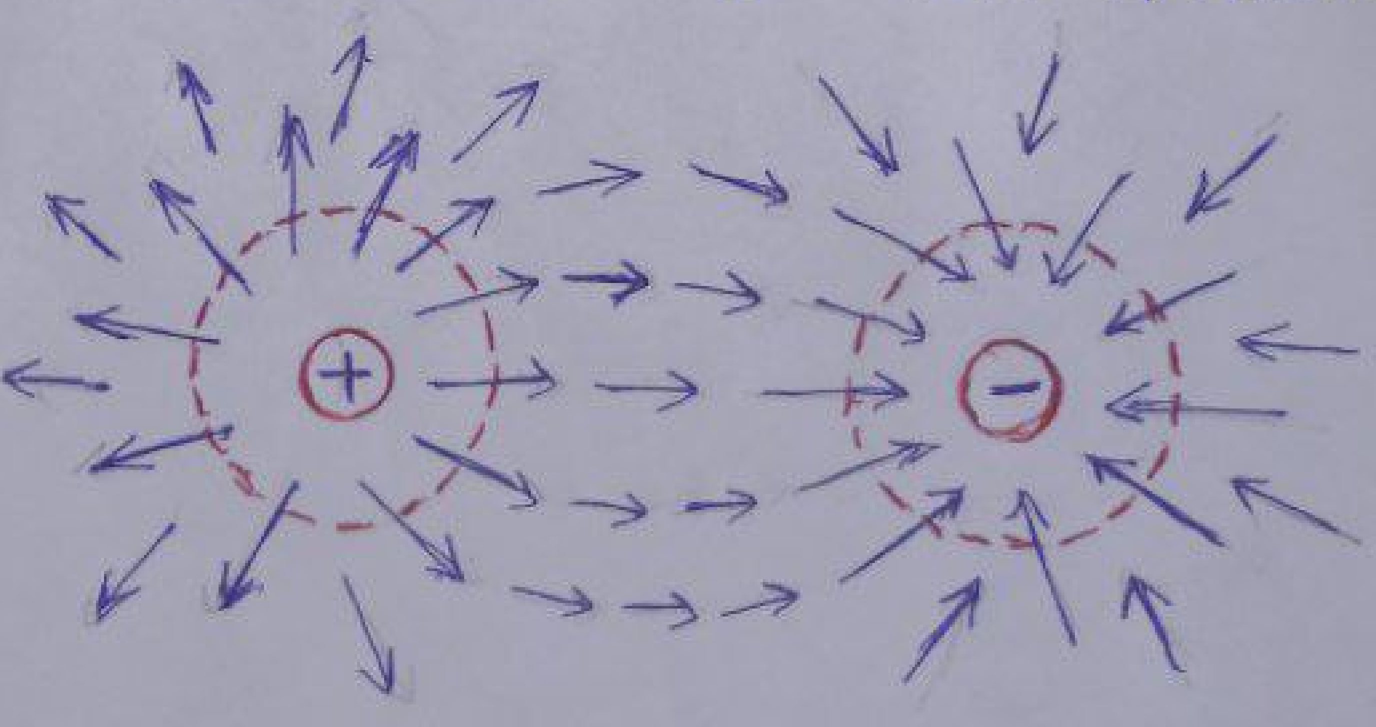
- The charge objects create electric field flow.

- Wherever there is an electric charge, there is an electric field & wherever there is an electric field, there is an electric charge.

- The spreading or flowing (spatial) an electric field is equal to an electric charge density in a certain medium.

- The divergence of electric field ($\vec{\nabla} \cdot \vec{E}$) means, the electric field lines spread away from the imaginary surface surrounding an electric charge. This spread



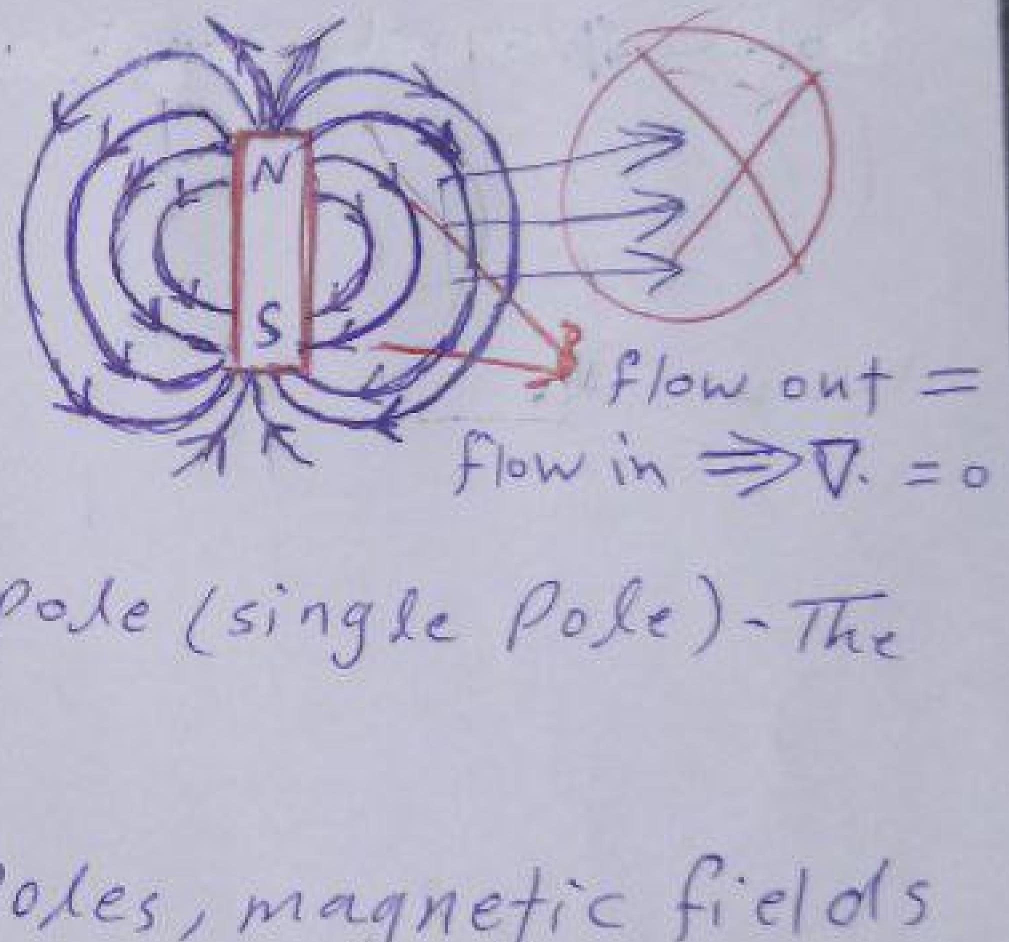


is spatial (spatial deravative), i.e., it is independent of time as we see that mathmatically:

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \Rightarrow \underbrace{\frac{\partial \vec{E}_x}{\partial x} + \frac{\partial \vec{E}_y}{\partial y} + \frac{\partial \vec{E}_z}{\partial z}}_{\text{from div. operator rules}} = \rho/\epsilon_0$$

$\boxed{2- \ \vec{\nabla}\cdot\vec{B} = 0}$ means,

- As a compare with first maxwell's equation before ($\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$), magnetic charge does not exist.
- Since $B = \mu H$, therefore, $\vec{\nabla}\cdot\vec{H}$ is also zero.
- There is no divergence of magnetic field (field don't flow in or out of any valume, that is, the same amount of flux enters as leaves (circular flow).

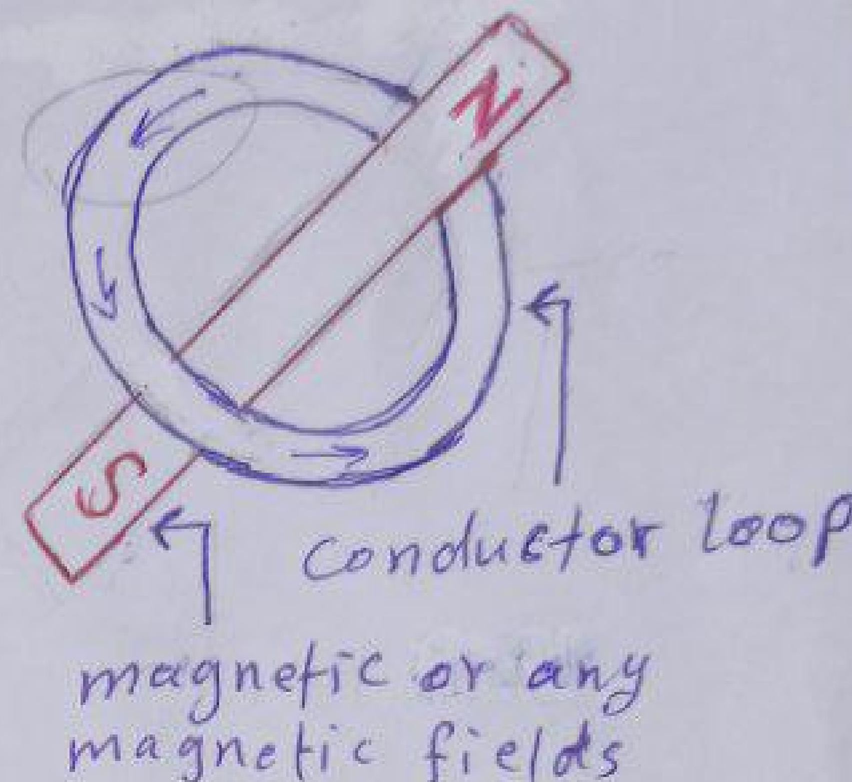


- There is no magnetic monopole (single Pole) - The magnetic is always dipole.
- Away from magnetic dipoles, magnetic fields flow in closed loop even for plane waves (have infinite radius loop).
- The deravative of magnetic field is also spatial deravative (changes with coordinates not with time), so

$$\vec{\nabla}\cdot\vec{B} = \frac{\partial \vec{B}_x}{\partial x} + \frac{\partial \vec{B}_y}{\partial y} + \frac{\partial \vec{B}_z}{\partial z} = 0$$

$\boxed{3-\ \vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}}$ means,

- $\vec{\nabla} \times \vec{E}$ means, there is conductor loop (conductor has free charges that are just stationary there). $\frac{\partial B}{\partial t}$ means, there is a magnetic field going throgh the loop. Any change in magnetic field with respect to time (strong or weak), it will cause the electric field to creat inside the conductor.



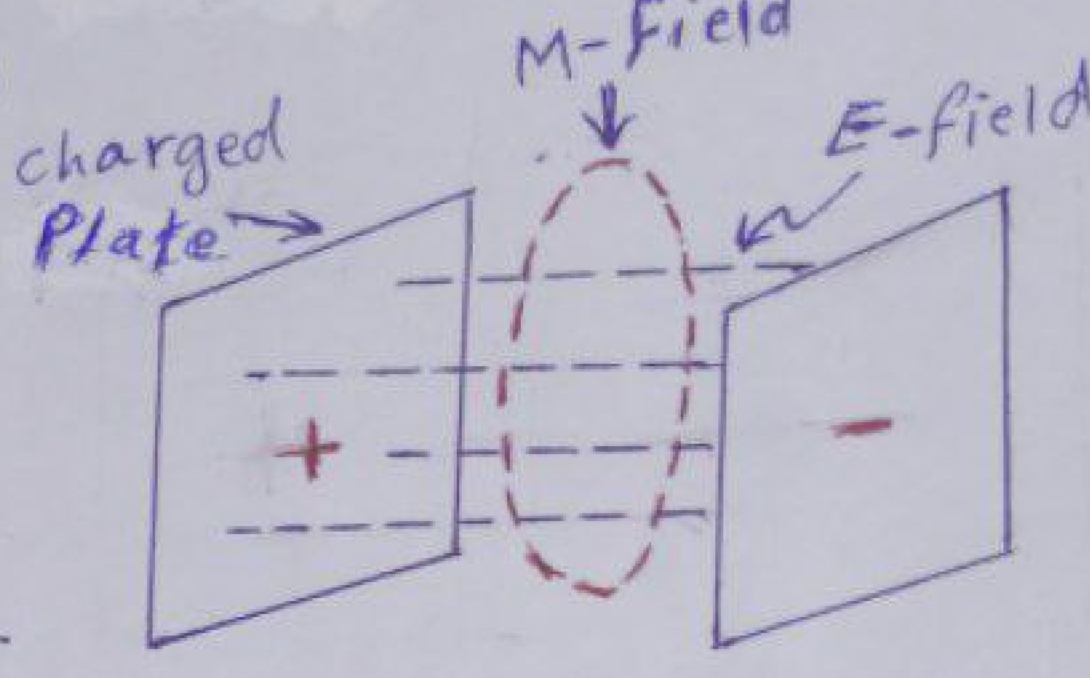
Or - Circulating electric field creats an a magnetic field changing in time.

- magnetic field changing in time creats or gives rise to an electric field circulating around it.

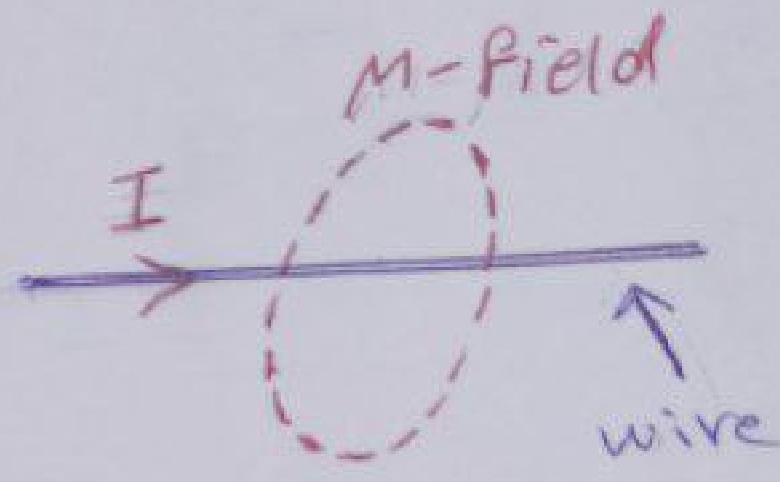
- The negative sign is just to make sure the current direction is right.

$$\boxed{4\text{-}\ \vec{\nabla} \times \vec{B} = -\frac{\partial}{\partial t}\vec{E} + j}$$

- The changing in the electric field with respect to time, there will be a magnetic field around it.

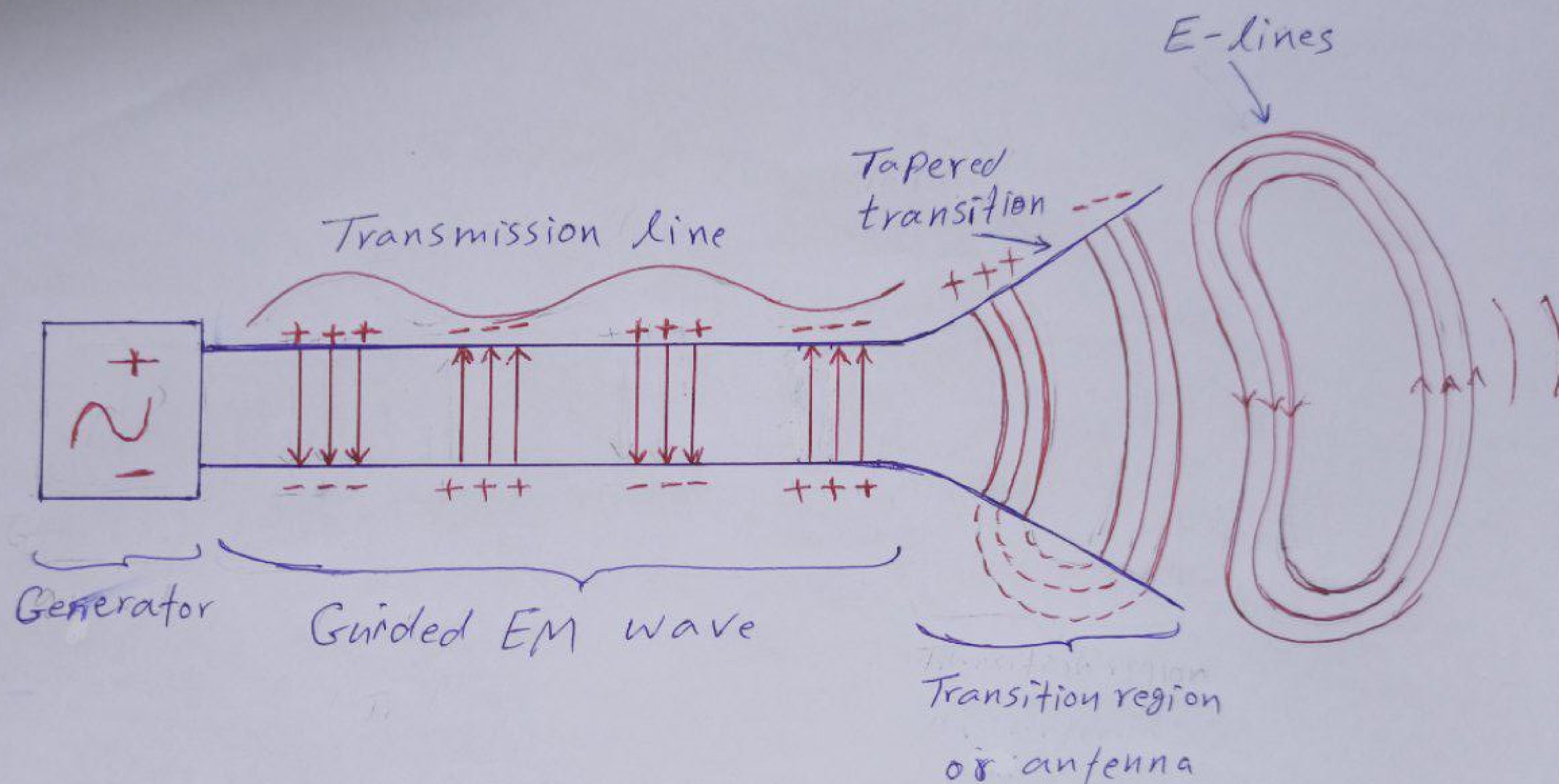


- If there is a wire with current, there will be a magnetic field around it & the strength of the field is proportional to the current in that wire.



Antenna mechanism

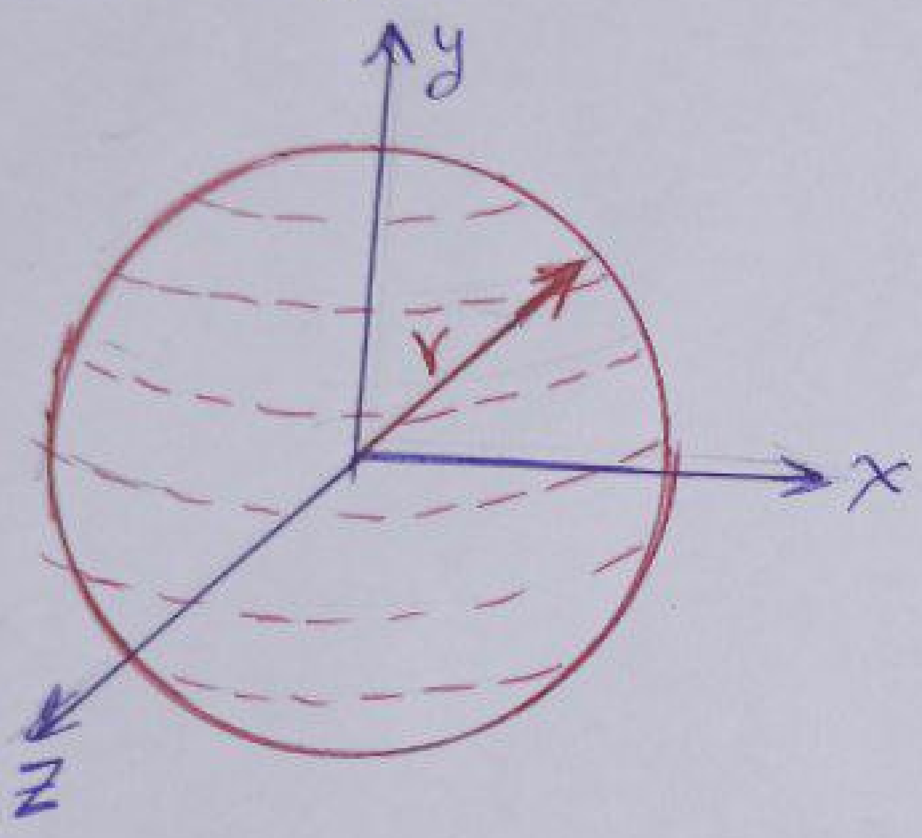
- Applying A-c source between the 2-conductors (the space between them is small fraction of wavelength) to produce a-c current.
- when the current flows, the free electric charges in conductors will divergent out of & in to each others (Maxwell's equation-1) Proportionally or accordingly to a-c current (sinsoidal) creating an electric fields a long transmission line.
- The taper in the end of T-L (antenna) tends to discontinuity which creats distribution in charges accelaration causing vibration in electric charges.
- This vibration or oscilation causes changing in electric field leading to appearence changed magnetic field around the electric field & perpendicular on it (maxwell's equation-4)
- These electric & magnetic field forms the electromagnitic waves propogating from antenn to free space when the taper appoaches the order of a wavelength or more.


E-lines
Tapered transition
Transmission line
Generator
Guided EM wave
Transition region or antenna

Antennas can be classified with respect to Radiation Pattern to :

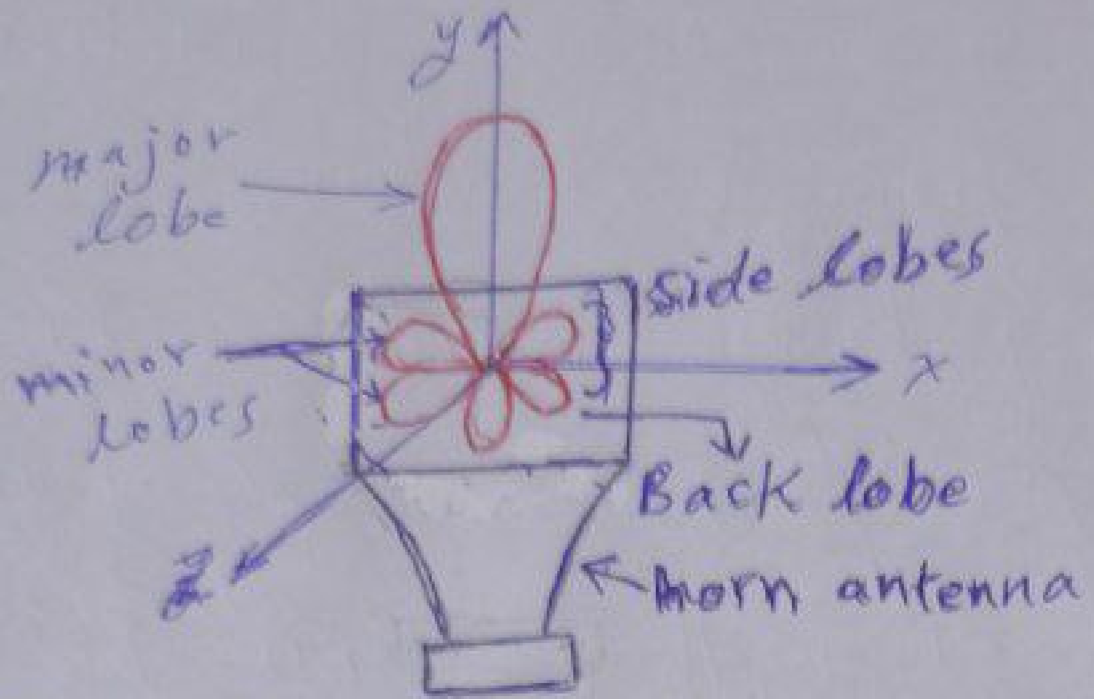
- Isotropic antenna
- Directional antenna
- Omnidirectional antenna

→ Isotropic antenna



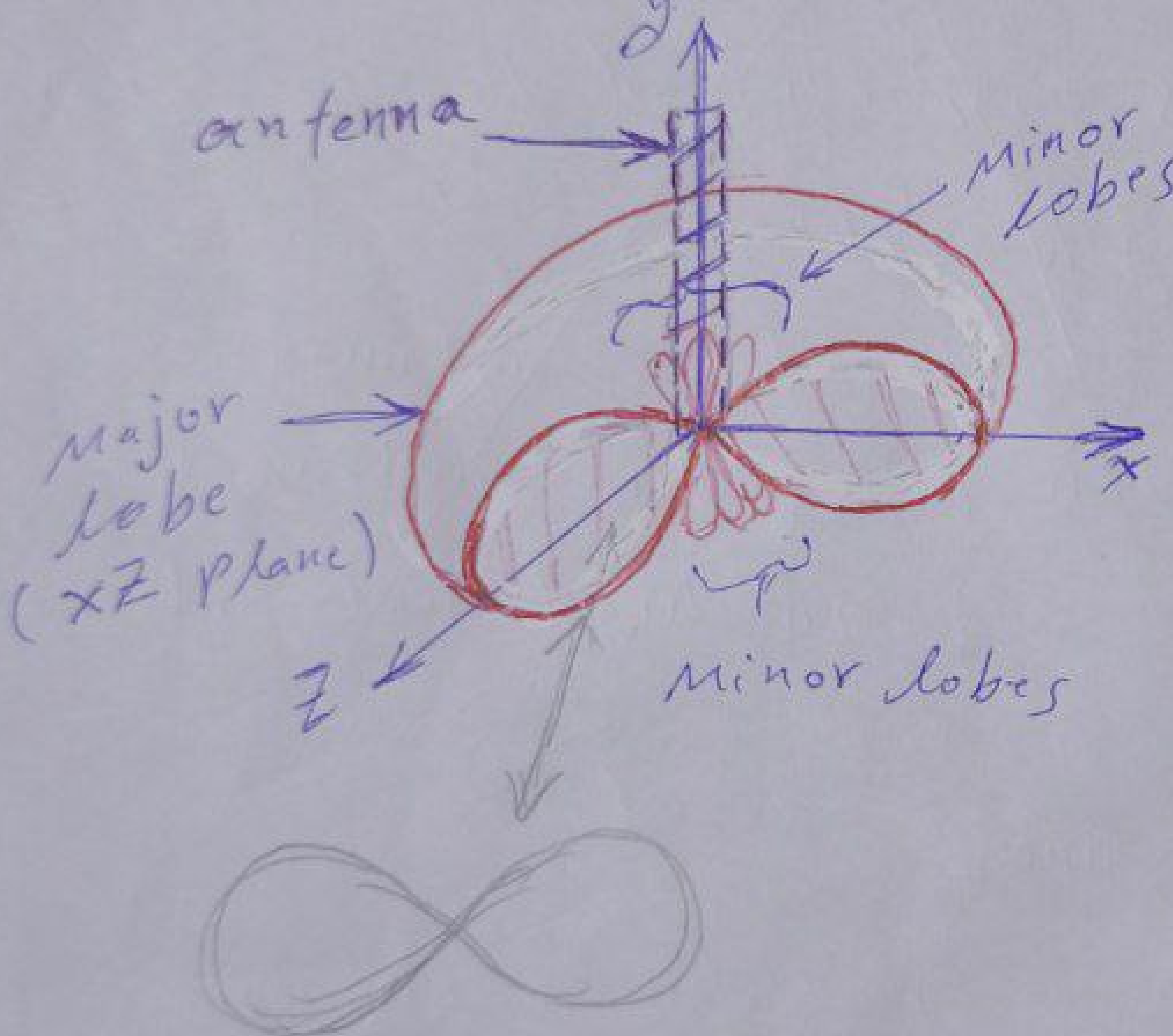
- radiates equally in all the direc.
- it's radiation Pattern is sphere
- hypothesis (not real) antenna

→ Directional antenna



- Directinal antenna radiates in particulare direction.
- ex- horn antenna

→ Omnidirectional antenna



- Omnidirectional antenna radates in plane
- Major lobe of this antenna is in plane
- In this antenna there is no back lobe

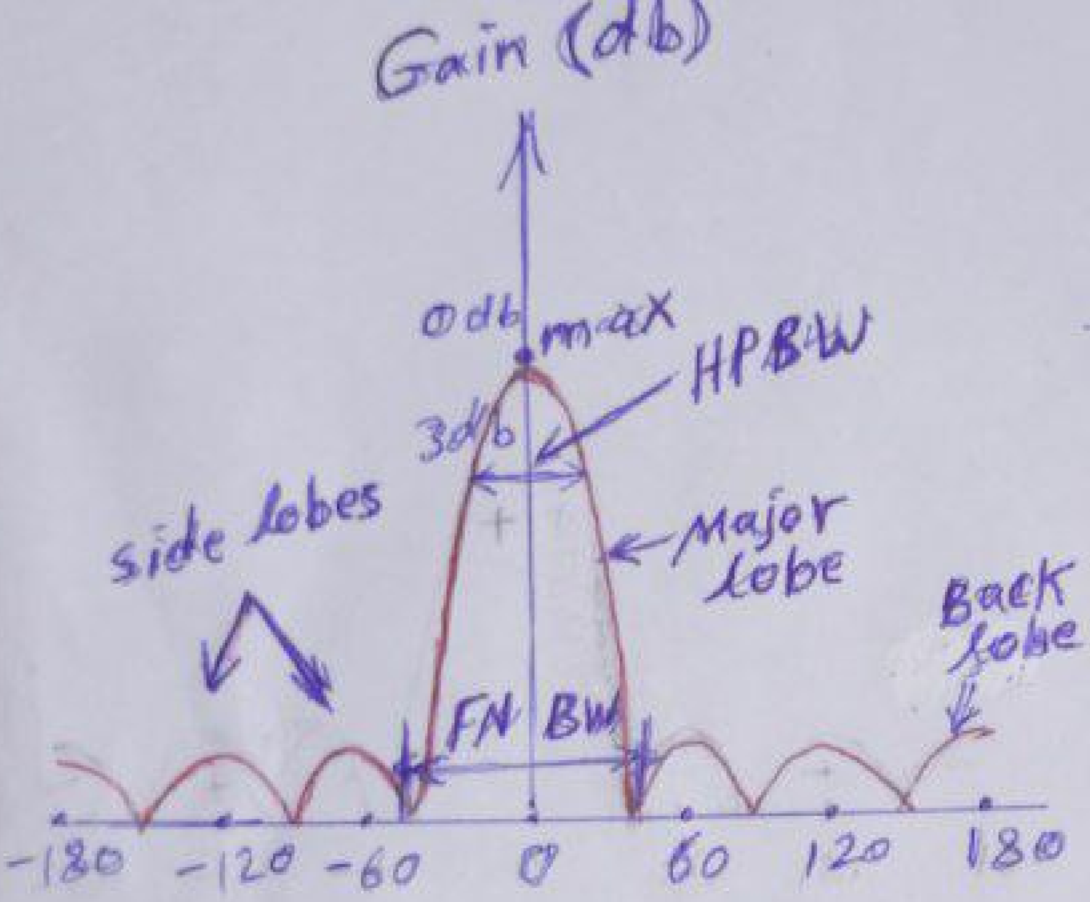
- ex/ Dipole antenn

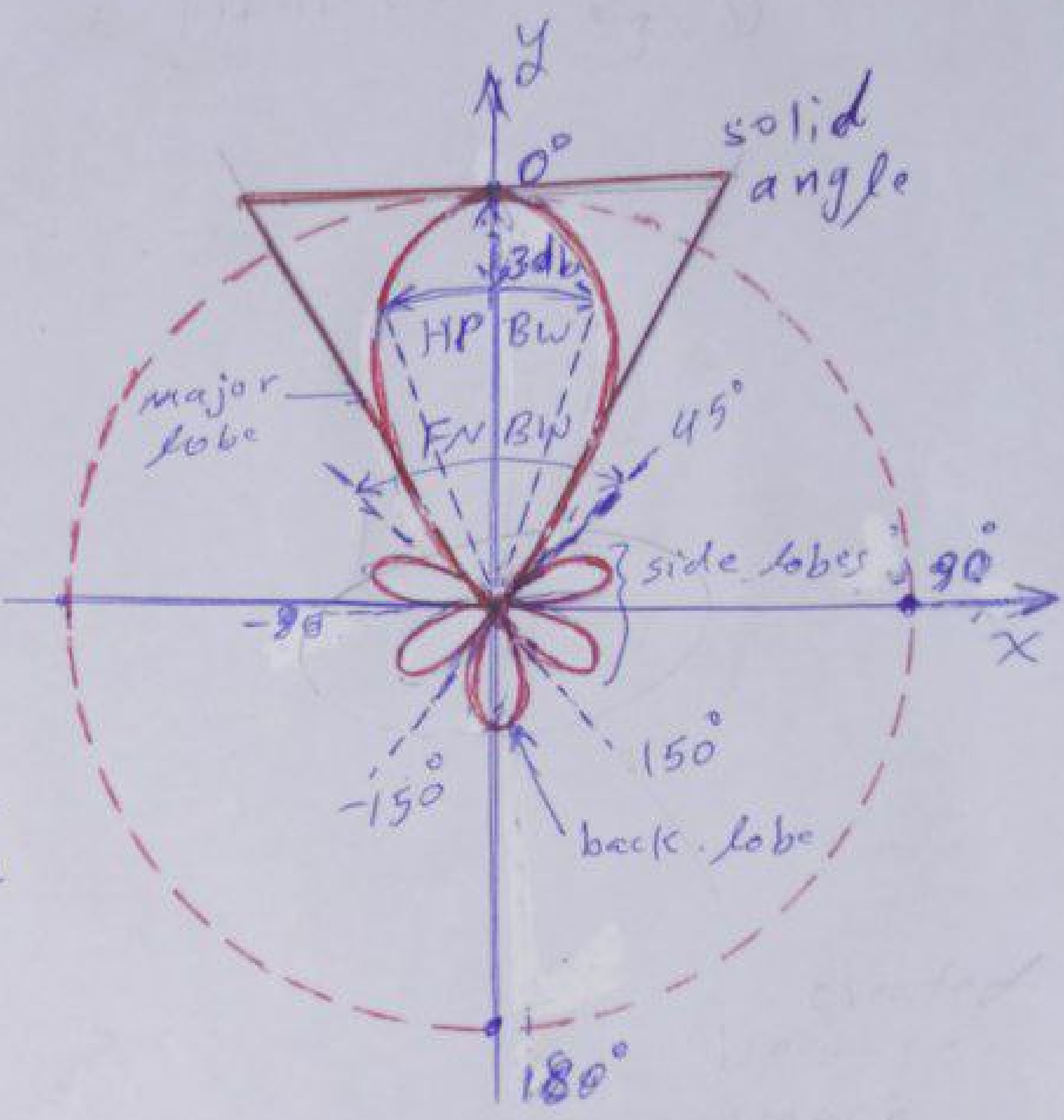
Fundimantial Antenna Parameteres

* Radiation Pattern
* Antenna gain
* Efficiency
* Antenna impedance
* Beam width
* Band width
* Antenna Temperature

Radiation pattern (Antenna pattern) is a mathmatical function [$E(\theta, \phi)$, $H(\theta, \phi)$] or a graphical representation of the radiation properties of the antenna [Radiation intensity, field strength $E(\theta, \phi)$, $H(\theta, \phi)$, directivity, Polarization, etc as a function of the angular space coordinaties (θ, ϕ).
So, graphically, radiation pattern can be plotted as a function of angular position & radial distance from the antenna, where the radiation pattern is determined in the far-field region.

Cartesian $\Longleftrightarrow$ Polar
(linear)





- HPBW (Half power Beam width),
  it is an angular width of major lobe, from maximum gain to 3 db down.
- FNBW (First Null Beam Width),
  it is a width of major lobe, where $E(\theta) = 0$
- Front to back ratio is a ratio of gain from major lobe to back lobe.

note θHPBW (mathmatically) is a Point where Power becomes half, i-e Power Pattern w.r.t θ =

$p = E^2(\theta) = \frac{1}{2} \Rightarrow$ amplitude pattern $= \frac{1}{\sqrt{2}} = 0.707$

جذر التربيعي $E(\theta)$

So, normalized Power Pattern in dB scale

$10 \log 1 = 0 \text{ dB}$,

$10 \log(\text{half Power}) = 10 \log 0.5 = -3 \text{ dB}$.

Example: An antenna has a field Pattern given by $E(\theta) = \cos^2\theta$ for $0° \leq \theta \leq 90°$. Find θ_{HPBW}.

Solution: At half Power $E(\theta) = 0.707$

$\because E(\theta) = \cos^2\theta$

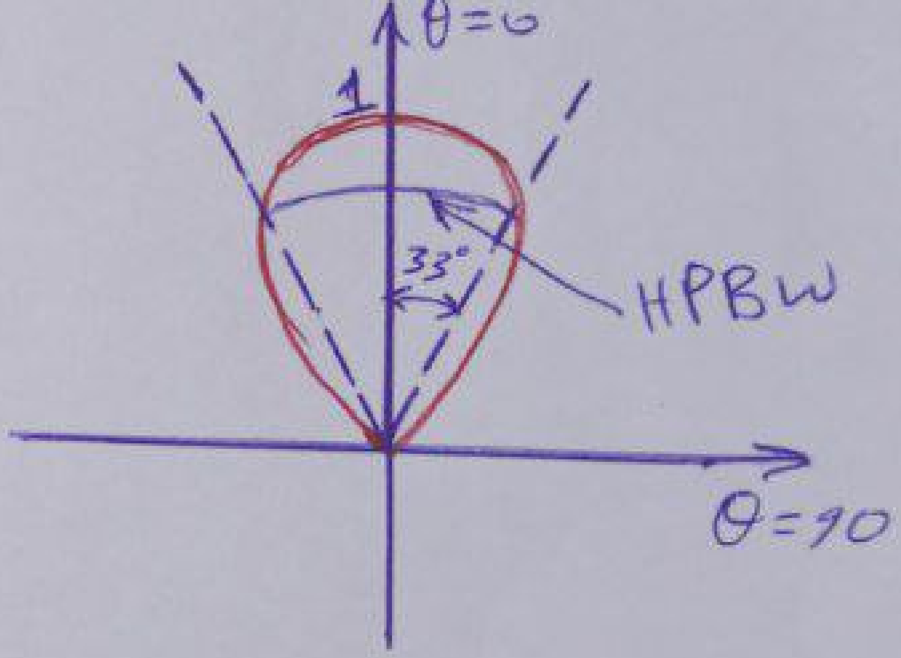
$\therefore \cos^2\theta = 0.707$

$\Rightarrow \cos\theta = \sqrt{0.707}$

$\Rightarrow \theta = \cos^{-1}\sqrt{0.707}$

$\Rightarrow \theta = 32.77 \simeq 33°$

$\Rightarrow HPBW = 2\theta = 2 * 33° = 66°$



while at FNBW $E(\theta) = 0$

so $\cos^2\theta = 0$

$\Rightarrow \theta = \cos^{-1} 0 = 90°$

$\therefore \theta_{FNBW} = 2\theta = 180°$

Ex.2: An antenna has a field Pattern given by $E(\theta) = \sin\theta$. Find HPBW & FNBW.

sol: At half Power:

$$E(\theta) = 0.707$$

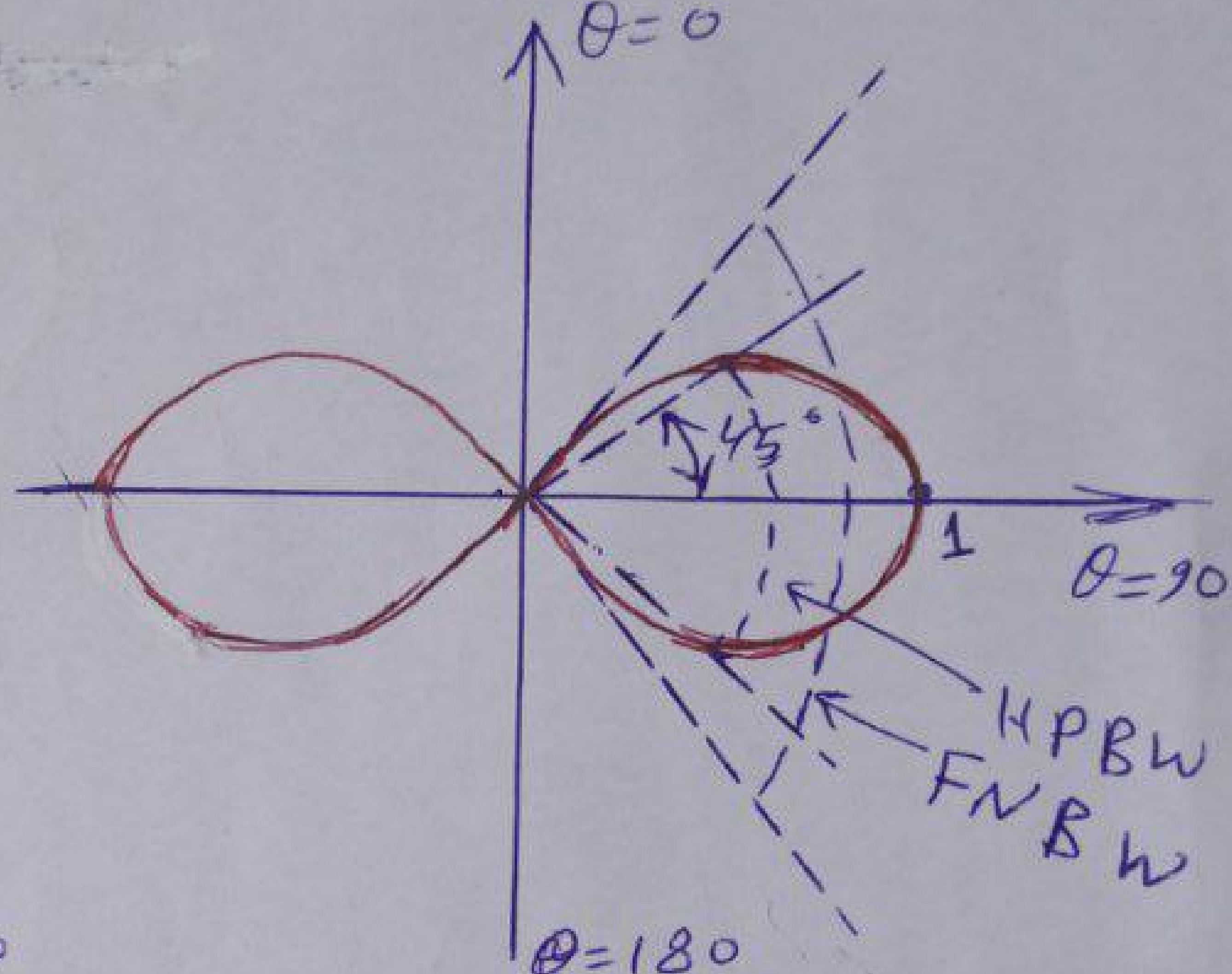
so $\sin^2\theta = 0.707$

$$\Rightarrow \sin\theta = \sqrt{0.707}$$

$$\Rightarrow \theta = \sin^{-1}\sqrt{0.707}$$

$$\Rightarrow \theta = 45^\circ$$

$$\Rightarrow HPBW = 2\theta = 90^\circ$$

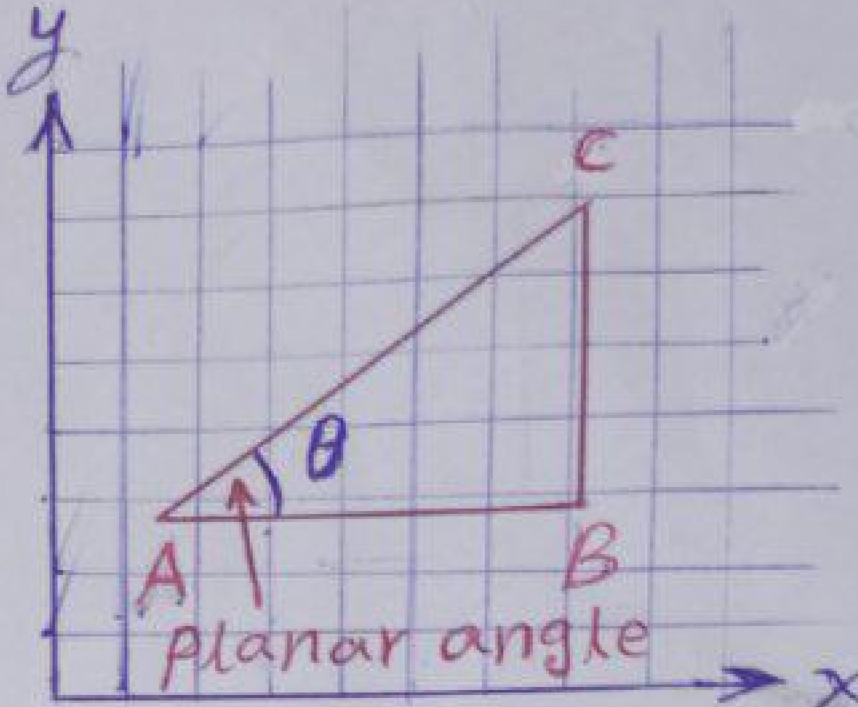


For FNBW: $E(\theta) = 0$

so $\sin\theta = 0 \Rightarrow \theta = 90^\circ$

$$\therefore FNBW = 2\theta = 180^\circ$$

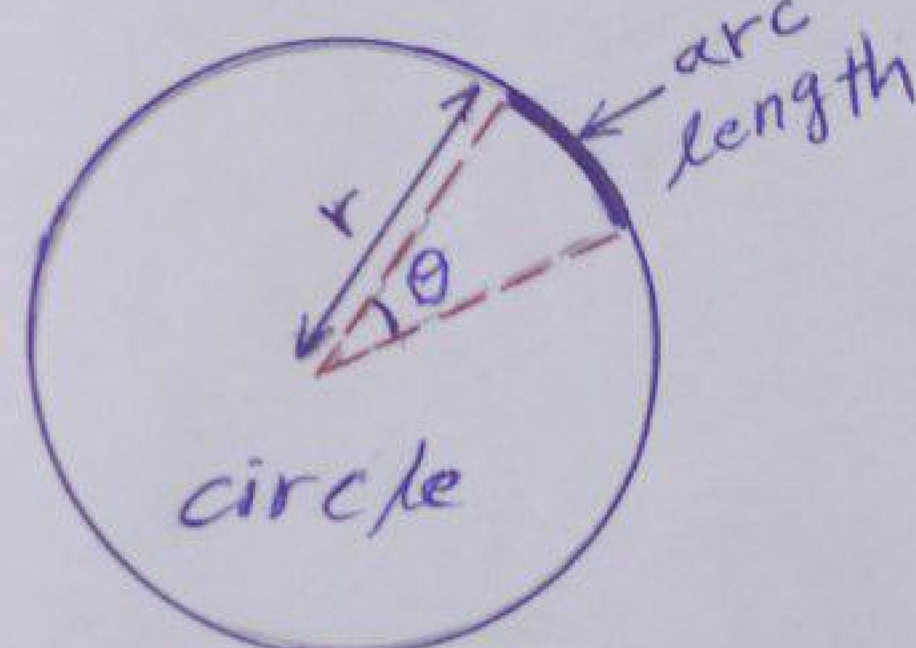
Plane angle : As it is known, the 2-dimintion is means a plane (2-d) & the angle in plane named plane angle or Planar angle so,



$$\tan\theta = \frac{opp}{adj} = \frac{BC}{AB} \Rightarrow$$

$$\theta = \tan^{-} \frac{BC}{AB} \text{ (deg)}$$

But, the angle in the circular surface (2-dim.),



is $\underset{\text{(in radian)}}{\theta} = \frac{\text{Arc Length}}{\text{radius}} = \frac{L}{r}$ (radian)

where 1 radian $= \frac{180}{\pi} \cong 57.3°$

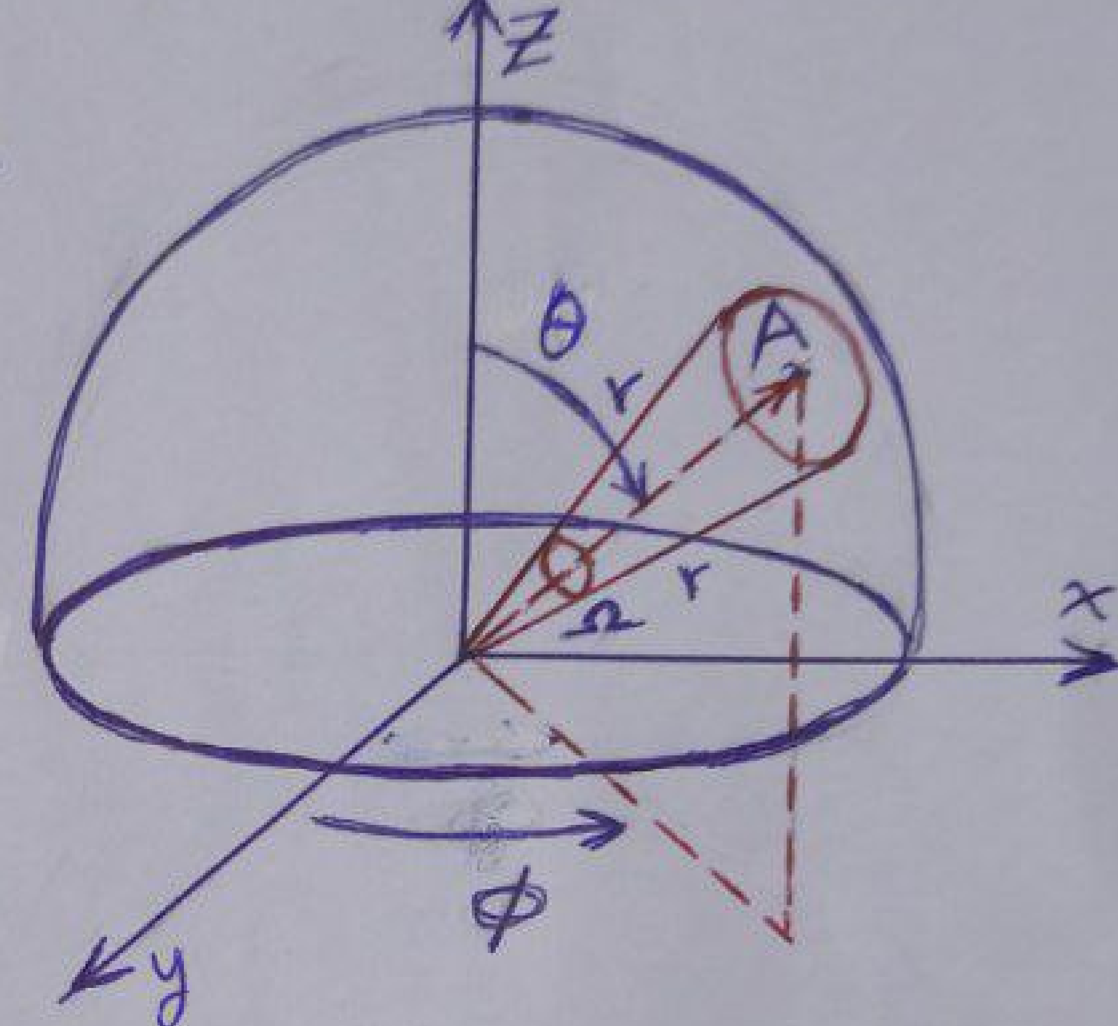
* Plane angle for complete circle is :

$$\underset{\text{(in radian)}}{\theta} = \frac{\text{arc length}}{\text{radius}} = \frac{\text{circumstances}}{r} = \frac{2\pi \cancel{r}}{\cancel{r}} =$$

$$2\pi = 2 * 3.14 = 6.28 \text{ radian} = 6.28 * 57.3 = 360°$$

note For small or tiny area, plane angle is $d\theta = \frac{dl}{r}$

solid angle : The angle which subtends an area of the surface of the sphere used to calculate howmuch of an object is in the field of view of another object.



So, solid angle $= \Omega$

$$= \frac{\text{Area}}{(\text{radius})^2}$$

$$= \frac{A}{r^2} \text{ steradian (sr)}$$

* $\text{steradian} = (\text{rad})^2$

* Solid angle represented in spherical coordinate system (Polar coordinate system), i-e, in 3-dimensions.

* Solid angle for complete sphere,

$$\Omega = \frac{4\pi r^2}{r^2} = 4\pi \text{ (sr)}$$

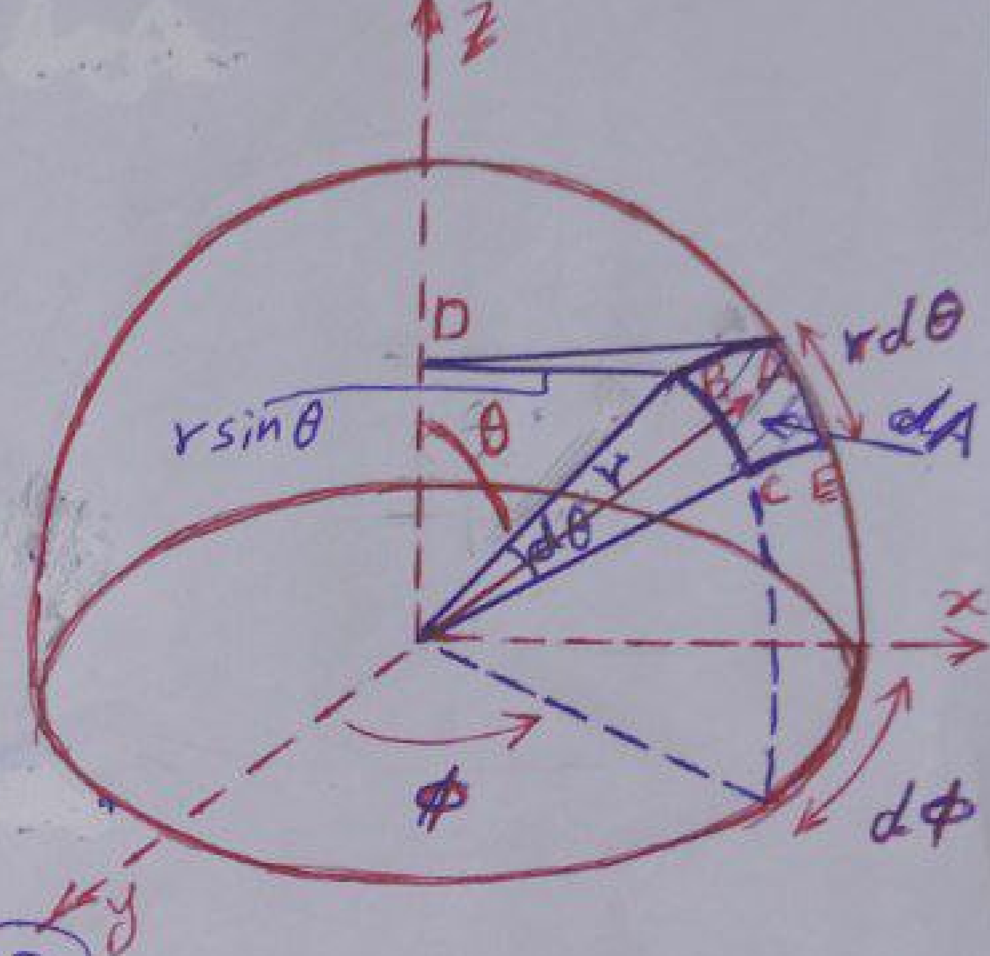
* For a tiny & infinitesimally small surface, the solid angle is $d\Omega = \frac{dA}{r^2}$,

where $dA \equiv$ small area $= AB \times BC$ ---- ①

By using the concept

befor $(\theta = \frac{L}{r}) \Rightarrow$

$L = r\theta$ ---------- ②



$\Rightarrow BC = r\,d\theta$ (from OBC traingle)

also, from the traingle OBD,

$\Rightarrow (BD \equiv AD) = r\sin\theta$ ---- ③

and by using the concept

$AB = r\sin\theta\, d\phi$ ----- ④

ultimately, by subtituting ③ & ④ in ① $\Rightarrow$

$dA = (r\sin\theta\, d\phi) \times (r\,d\theta) = r^2 \sin\theta\, d\theta\, d\phi$

so, $d\Omega = \frac{dA}{r^2} = \frac{\cancel{r^2} \sin\theta\, d\theta\, d\phi}{\cancel{r^2}} = \sin\theta\, d\theta\, d\phi$ --- ⑤

where θ angle is zenith (or polar) angle

ϕ angle is azimuthal angle.